Lesson 22: Determining Control Stability Using Bode Plots

1

ET 438A AUTOMATIC CONTROL SYSTEMS TECHNOLOGY

lesson22et438a.pptx

Learning Objectives

2

After this presentation you will be able to:

- List the control stability criteria for open loop frequency response.
- Identify the gain and phase margins necessary for a stable control system.
- Use a Bode plot to determine if a control system is stable or unstable.
- Generate Bode plots of control systems the include dead-time delay and determine system stability.

lesson22et438a.pptx

Bode Plot Stability Criteria

3

Stable Control System	Open loop gain of less than 1 (G<1 or G<0dB) at open loop phase angle of -180 degrees
Oscillatory Control System Marginally Stable	Open loop gain of exactly 1 (G=1 or G= 0dB) at open loop phase angle of -180 degrees
Unstable Control System	Open loop gain of greater than 1 (G>1 or G>0dB) at open loop phase angle of -180 degrees

lesson22et438a.pptx

Phase and Gain Margins

4

Inherent error and inaccuracies require ranges of phase shift and gain to insure stability.

Gain Margin – Safe level below 1 required for stability Minimum level : G=0.5 or -6 dB at phase shift of 180 degrees

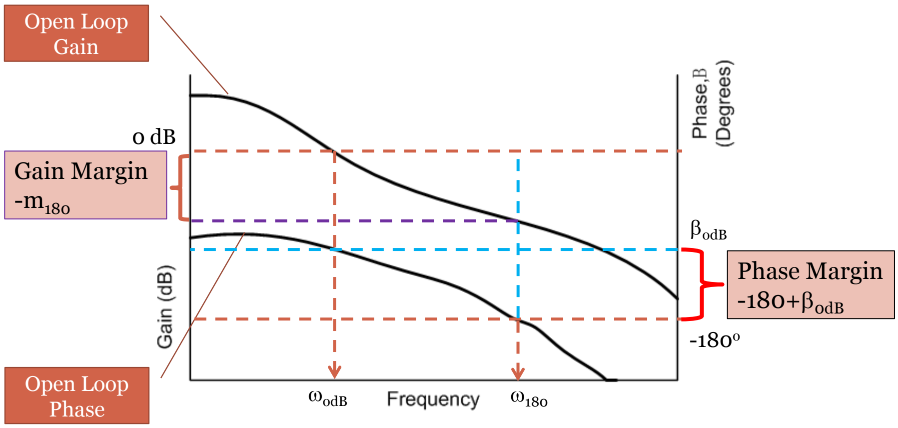
Phase Margin – Safe level above -180 degrees required for stability Minimum level : ϕ=40 degree or -180+ 40=-140 degrees at gain level of 0.5 or 0 dB.

lesson22et438a.pptx

Determining Phase and Gain Margins

5

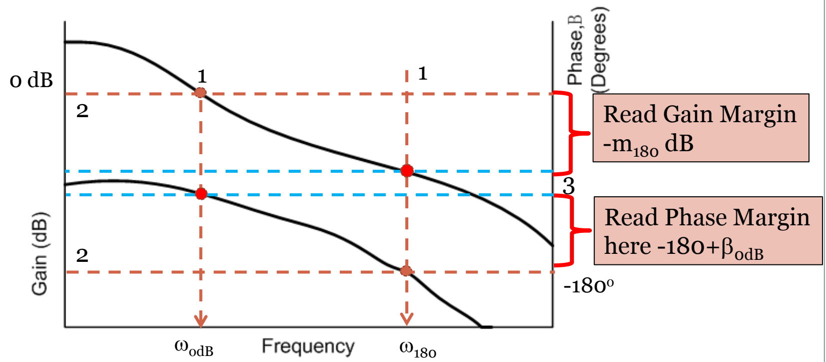
Define two frequencies: ω_{0dB} = frequency of 0 dB gain
ω_{180} = frequency of -180 degree phase shift



Determining Phase and Gain Margins

6

Procedure: 1) Draw vertical lines through 0 dB on gain and -180 on phase plots.
2) Draw horizontal lines through 0 dB and -180 so that they intersect with the vertical lines.
3.) Draw two more horizontal lines that intersect the -180 line on the gain plot and the 0 dB line on the phase plot.



Stability Analysis Using Bode Plots

7

Bode plot stability analysis is idea for systems with dead-time delay. Delay represented by phase shift that increases with frequency.

Example 22-1: A first order lag process has a dead-time delay of 2 seconds and is controlled by a proportional controller. The open loop transfer function is:

$$GH(s) = 40 \cdot \left[\frac{1}{1+100s}\right] \cdot e^{-2s}$$

1) Find the magnitude and phase angle of the transfer function at the following frequencies: ω=0.001, 0.01, 0.1 and 1 radian/sec using hand calculations.
2) Use MatLAB and construct the Bode plots of the system and then determine the gain and phase margin of the system.

lesson22et438a.pptx

Example 22-1 Solution (1)

8

Substitute jω=s $\quad GH(j\omega) = 40 \cdot \left[\frac{1}{1+100j\omega}\right] \cdot e^{-2j\omega}$

Where $\quad e^{-2j\omega} = \begin{cases} G = 1 \text{ for all } \omega \\ \phi = -2 \cdot (57.6) \cdot \omega \end{cases}$

For jω=j0.001 $\quad \left[\frac{1}{1+100j0.001}\right] = \left[\frac{1}{1+j0.1}\right] = \frac{1}{1.005\angle 5.71^\circ} = 0.995\angle -5.71^\circ$

$$e^{-2j0.001} = \begin{cases} G = 1 \text{ for all } \omega \\ \phi = -2 \cdot (57.6) \cdot 0.001 \end{cases} = 1\angle -0.115^\circ$$

$$|GH(j0.001)| = 40 \cdot 0.995 \cdot 1 = 39.8$$

$$\text{ang}[GH(j0.001)] = -5.71^\circ + (-0.115^\circ) = -5.825^\circ$$

lesson22et438a.pptx

Example 22-1 Solution (2)

9

For jω=j0.01

$$GH(j0.01)=\left[\frac{1}{1+100j0.01}\right]=\left[\frac{1}{1+j1}\right]=\frac{1}{1.414\angle 45^\circ}=0.707\angle -45^\circ$$

$$e^{-2j0.01}=\begin{cases} G=1 \text{ for all } \omega \\ \phi=-2\cdot(57.6)\cdot 0.01 \end{cases}=1\angle -1.15^\circ$$

$$|GH(j0.01)|=40\cdot 0.707\cdot 1=28.28$$

$$\text{ang}[GH(j0.01)]=-45^\circ+(-1.15^\circ)=-46.15^\circ$$

For jω=j0.1

$$GH(j0.1)=\left[\frac{1}{1+100j0.1}\right]=\left[\frac{1}{1+j10}\right]=\frac{1}{10.05\angle 84.3^\circ}=0.0995\angle -84.3^\circ$$

Example 22-1 Solution (3)

10

For jω=j0.1 cont.

$$e^{-2j0.1}=\begin{cases} G=1 \text{ for all } \omega \\ \phi=-2\cdot(57.6)\cdot 0.1 \end{cases}=1\angle -11.52^\circ$$

$$|GH(j0.1)|=40\cdot 0.0995\cdot 1=3.98$$

$$\text{ang}[GH(j0.1)]=-84.3^\circ+(-11.52^\circ)=-95.82^\circ$$

For jω=j1

$$GH(j1.0)=\left[\frac{1}{1+100j1}\right]=\left[\frac{1}{1+j100}\right]=\frac{1}{100\angle 89.4^\circ}=0.01\angle -89.4^\circ$$

$$e^{-2j0.1}=\begin{cases} G=1 \text{ for all } \omega \\ \phi=-2\cdot(57.6)\cdot 1 \end{cases}=1\angle -115.2^\circ$$

$$|GH(j0.1)|=40\cdot 0.01\cdot 1=0.4 \quad \text{ang}[GH(j0.1)]=-89.4^\circ+(-115.2^\circ)=-204.6^\circ$$

Example 22-1 Solution (4)

11

Calculation summary

Convert all magnitudes to decibels

$$GH(j0.001)_{dB} = 20\log(39.8) = 32\ dB$$
$$GH(j0.01)_{dB} = 20\log(28.28) = 29\ dB$$
$$GH(j0.1)_{dB} = 20\log(3.98) = 12\ dB$$
$$GH(j1)_{dB} = 20\log(0.4) = -8.0\ dB$$

Frequency (rad/sec)	GH	GH (dB)
0.001	$39.8\angle -5.83^\circ$	$32\angle -5.83^\circ$ dB
0.01	$28.28\angle -46.15^\circ$	$29\angle -46.15^\circ$ dB
0.1	$3.98\angle -95.82^\circ$	$12\angle -95.82^\circ$ dB
1.0	$0.4\angle -204.6^\circ$	$-8\ \angle -204.6^\circ$ dB

Example 22-1 Solution (5)

12

Construct an open-loop Bode plot using MatLAB and find the gain and phase margins for the control system. Example code follows:

```
clear all;
close all;
numgh=[40];                     % define the forward gain numerator and denominator
                                coefficients
demgh=[100 1];
Gh=tf(numgh,demgh);             % construct the transfer function
[m p w]=bode(Gh,{0.001,1});     % Use the bode function with its arguments so that it returns the
                                % magnitude, m, the phase shift, p and the frequencies so that
                                % the effect of the dead time delay can be added to the system
                                % now compute the values of phase shift for the time delay using
                                % the formula -2*w*57.6
pd=-2*w*57.6;
```

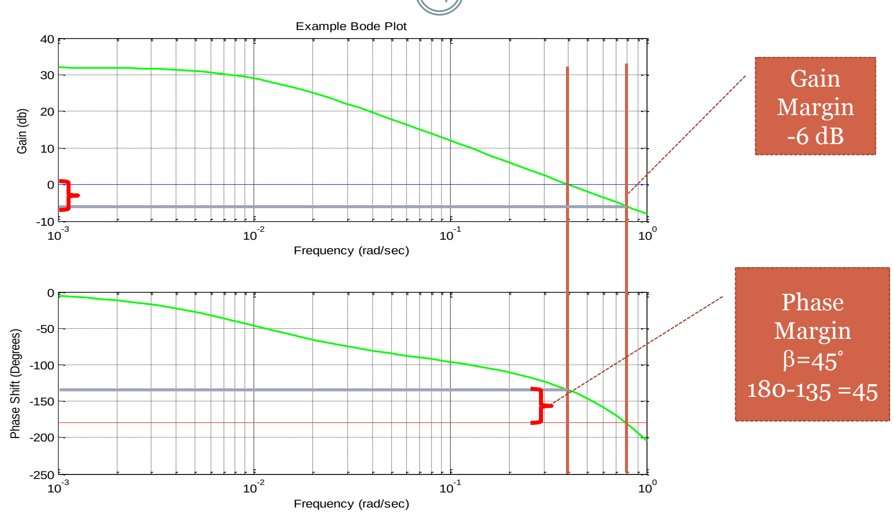
Example 22-1 Solution (6)


```
% Add the phase shift of the transfer function to the dead-time delay
% take the phase shift out of the 3 column array [m p w]

phase=p(:);
pt=pd+phase;
db=20.*log10(m);   % compute the gain in db

figure;            % create a figure window
subplot(2,1,1);    % divide the plot area in two parts
semilogx(w,db,'go-');  %plot gain in dB on a semilog x-axis
xlabel('Frequency (rad/sec)');  % add labels and title. Turn on the grid.
ylabel('Gain (db)');
title('Example Bode Plot');
grid on;
subplot(2,1,2);  % now do the same for the phase shift plot
semilogx(w,pt,'go-');
xlabel('Frequency (rad/sec)');
ylabel('Phase Shift (Degrees)');
grid on
```


Example 22-1 Solution (7)



End Lesson 22: Determining Control Stability Using Bode Plots

15

ET 438A AUTOMATIC CONTROL SYSTEMS TECHNOLOGY

lesson22et438a.pptx